

# Description of Quantum Systems by Random Matrix Ensembles of High Dimensions

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### Abstract

The new Theorem on location of maximum of probability density functions of dimensionless second difference of the three adjacent energy levels for  $N$ -dimensional Gaussian orthogonal ensemble  $GOE(N)$ ,  $N$ -dimensional Gaussian unitary ensemble  $GUE(N)$ ,  $N$ -dimensional Gaussian symplectic ensemble  $GSE(N)$ , and Poisson ensemble  $PE$ , is formulated: *The probability density functions of the dimensionless second difference of the three adjacent energy levels take on maximum at the origin for the following ensembles:  $GOE(N)$ ,  $GUE(N)$ ,  $GSE(N)$ , and  $PE$ , where  $N \geq 3$ . The notions of level homogenization with level clustering and level homogenization with level repulsion are introduced.*

Many complex  $N$ -level quantum systems exhibiting universal behaviour depending only on symmetry of Hamiltonian matrix of the system are divided into: Gaussian orthogonal ensemble  $GOE(N)$ , or Gaussian unitary ensemble  $GUE(N)$ , or Gaussian symplectic ensemble  $GSE(N)$ . The Gaussian ensembles are used in study of quantum systems whose classical-limit analogs are chaotic. The Poisson ensemble  $PE$  (Poisson random-sequence spectrum) is composed of uncorrelated and randomly distributed energy levels and it describes quantum systems whose classical-limit analogs are integrable. The standard statistical measure is Wigner’s distribution of the  $i$ th nearest neighbour spacing:

$$s_i = \Delta^1 E_i = E_{i+1} - E_i, \quad i=1, \dots, N-1. \quad (1)$$

For  $i$ th second difference (the  $i$ th second differential quotient) of the three adjacent energy levels:

$$\Delta^2 E_i = \Delta^1 E_{i+1} - \Delta^1 E_i = E_i + E_{i+2} - 2E_{i+1}, \quad i=1, \dots, N-2, \quad (2)$$

we calculated distributions for GOE(3), GUE(3), GSE(3), and PE Refs [1–3].

We formulate the following

**Theorem:** *The probability density functions of the dimensionless second difference of the three adjacent energy levels take on maximum at the origin for the following ensembles: GOE(N), GUE(N), GSE(N), and PE, where  $N \geq 3$ .*

We present the idea of proof. For Gaussian ensembles it can be shown that second difference distributions are symmetrical functions for  $N \geq 3$ . Hence, the first derivatives of the distributions at the origin vanish. For Poisson ensemble the second difference distribution is Laplace one for  $N \geq 3$ . Therefore, the distribution takes on maximum at zero.

The inferences are the following:

1. The quantum systems show tendency towards the homogeneity of levels (equal distance between adjacent levels). We call it *homogeneization of energy levels*.
2. There are two generic homogeneizations: the first is typical for Gaussian ensembles, the second one for Poisson ensemble. For the former ensembles we define *level homogenization with level repulsion* as follows. Energy levels are so distributed that the situation that both the spacings and second difference vanish:

$$\Delta^2 E_i = s_i = s_{i+1} = 0, \quad (3)$$

is the most probable one. For the latter ensemble *level homogenization with level clustering* is described below. Now it is the most probable that only the second difference is equal to zero but the two nearest neighbour spacings are nonzero:

$$\Delta^2 E_i = 0, s_i = s_{i+1}, s_i \neq 0. \quad (4)$$

3. The assumption of non-zero value by the second difference is less probable than the assumption of zero value. Equivalently, the inequality of the two nearest neighbour spacings is less probable than their equality.
4. The predictions of the Theorem are corroborated by numerical and experimental data Refs [1–3].

The theorem could be extended to other ensembles, *e.g.* circular ones, and it is a direction of future development.

## REFERENCES

- [1] M. M. Duras and K. Sokalski, Phys. Rev. E **54**, 3142 (1996).
- [2] Maciej M. Duras, *Finite difference and finite element distributions in statistical theory of energy levels in quantum systems*, Ph. D. thesis, Jagellonian University, Cracow, July 1996.
- [3] M. M. Duras and K. Sokalski, *Physica* **D125**, 260 (1999).